(Mis)information diffusion and the financial market *

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Abstract

This paper investigates the interplay between information diffusion in social networks and its impact on financial markets with an agent based model (ABM). Agents receive and exchange information about an observable stochastic component of the dividend process of a risky asset à la Grossman and Stiglitz (1980). A small proportion of the network has access to a private signal about the component, which can be clean (information) or distorted (misinformation). Other agents are uninformed and can receive information only from their peers. All agents are Bayesian in updating their beliefs, but they are so in a behavioural way, so that in the construction of the likelihood function, they replace true precision with an individual parameter which depends on an endogenous and time evolving measure of the agent confidence in the source of the information. We examine, by means of simulations, how information diffuses in the network and provide a framework to account for delayed absorption of shocks, that are not immediately priced as predicted by classical financial models. We show the effect of the network topology on the resulting asset price and offer an interpretation for excess volatility with respect to fundamentals, persistence amplification and lepto-kurtosis of returns.

***JEL Classification:**D53; D82; D85; G12; G41*

1 Introduction

The efficient-market hypothesis states that financial markets are efficient. Prices reflect all publicly available information about an asset. If a new source of information becomes available, investors will trade on it and push the price towards the new efficient level. According to models with a representative Rational Agent as introduced in Muth (1961), information should be readily available to agents when forming their expectations. This implies that as soon as a new piece of information is released each market participant should immediately incorporate it in its expectation formation process. It seems unlikely however that everyone can access perfectly all sources of information and even more so that all agents simultaneously and independently receive and process it. In this paper we are going to build an agent based model with parsimonious relaxations to rationality in two dimensions. We will introduce heterogenous access to information and allow

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for delayed transmission news to some agents by explicitly modelling the social network structure in which individuals are connected. These deviations from rationality for which there is ample empirical evidence, will be enough to qualitatively reproduce stylized facts of financial markets, that cannot be captured in a rational setting and that other models of bounded rationality are able to match only by introducing zero intelligence or backward looking individuals. In theory, anyone can process information, but the ability to effectively process and analyze information varies greatly between individuals. An individual's ability to process information can be influenced by a variety of factors, including their level of education, experience, cognitive abilities, and personal interests. For example, someone with a background in finance may be better equipped to process and analyze financial data than someone without this background. It seems therefore reasonable to assume that individuals observing a certain signal or news, might form incorrect beliefs. These beliefs can lead to misinformation about an asset being priced. If misinformation about an asset is spread and believed by a large number of investors, it has the potential to affect demand for the asset and thus its price. The drivers for information diffusion are generally traditional media outlets and regulated disclosure. However, social networks, particularly for retail investors, can play a role in diffusion because they enable individuals and organizations to quickly and easily share information with a large audience.

The literature provides multiple examples of inefficency of financial markets. Huberman and Regev (2001) show that stock prices of a company, CASI Pharmaceuticals did not incorporate new information for five months. They point out that news was initially released as a research article in the journal Nature, but investors reacted only when a Wall Street Journal article reposted the findings of the original study. In other worlds, there seems to be some room for asymmetry of information in financial market. A first possible explanation for this asymmetry is simply that not all investors have access to information. The implication of insider trading on financial markets has been analyzed theoretically by Kyle (1985), Benabou and Laroque (1992) and Collin-Dufresne and Fos (2016). The main findings are that the speed of incorporation of news into a financial market might vary since access to private information gives incentive to manipulate the market. In practice however insider trading is generally prohibited in many countries and therefore not a satisfactory explanation. Another source of asymmetry can be represented by behavioural components. In the Huberman and Regev (2001) case, even though information was publicly available, some investors might lack the knowledge or the capacity to process it. Dellavigna and Pollet (2009) offer evidence of limited attention by showing that investors are less reactive to news on Fridays and show how to construct a strategy exploiting the underreaction to information caused by limited attention. Another strand of research has focused on the impact of misinformation. Clarke et al. (2021) show that fake news has a direct impact on retail trading and prices.

The rest of the paper is structured as follows. Section 2 describes the model, focusing on the two blocks that constitute the ABM, the financial market block and the information diffusion block. Section 3 presents numerical simulations and results. Section 4 concludes.

2 Model

2.1 The financial market

The market is based on Grossman and Stiglitz (1980). There is an economy with I consumers, indexed by $i = (1, 2, \dots, \mathrm{I})$. Consumers are infinitely lived and at the beginning of every period they receive the same endowment W_0. They have the same utility over the end of period wealth, given by

$$U(W_t) = -e^{-aW_t}, \tag{1}$$

where $a > 0$ is the coefficient of risk aversion. In order to transfer wealth from the beginning of the period to the end, there are two types of security: a risk-free and a risky asset. We define by p_t the price of the risky asset in a generic time t and normalize the price of the risk-free asset to 1. In every period consumers decide how to allocate their initial endowment, choosing between the two possible securities. At the end of the period they receive profits based on their portfolio, and immediately consume their wealth. Given their participation in the financial market, throughout the paper we will use interchangeably the terms consumers, investor and agent. Defining $X_{i,t}$ as the consumer's demand of the risky asset and $M_{i,t}$ as the demand of the safe asset, the allocation choice is subject to the budget constraint

$$p_t X_{i,t} + M_{i,t} = W_0. \tag{2}$$

The risk-free rate is $R > 1$ and the risky asset pays a stochastic payoff which is equal to a dividend claim plus the future price of the asset

$$y_{t+1} = p_{t+1} + d_{t+1}. \tag{3}$$

The presence of the future price in the right-hand side of equation (3) ensures a positive feedback mechanism of expectations. This is a well established feature of financial markets and documented among others by Heemeijer et al. (2009) and a feature of our work absent in the original Grossman and Stiglitz (1980) model and more recent works like that of Gerotto, Pellizzari and Tolotti (2019). There are two stochastic components determining the realization of future dividends

$$d_{t+1} = d + \theta_{t+1} + \varepsilon_{t+1}, \tag{4}$$

with $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ being pure unobservable noise. The stochastic component θ_{t+1} is a partially observable component of dividends, and evolves according to

$$\theta_{t+1} = \beta\theta_t + \eta_{t+1}, \tag{5}$$

with $\beta \in (0, 1)$, $\eta_{t+1} \sim \mathcal{N}(0, \sigma_\eta^2)$ and ε_{t+1} and η_{t+1} being independent. This implies that fundamentals regarding the risky asset are stochastic but that some information about them is revealed in advance. However this information is not immediately incorporated in the asset price. The end of period t wealth for the i^{th} consumer is given by

$$W_{i,t} = RM_{i,t} + y_{t+1}X_{i,t} = R(W_0 - p_t X_{i,t}) + y_{t+1}X_{i,t}. \tag{6}$$

Agents optimize their end of period wealth, which given the normality of y_{t+1} is also normal. The optimization problem is therefore given by

$$\max_{\{X_{i,t}\}} \left(-exp \left\{ -a\mathbb{E}_{i,t}(W_{i,t}) + \frac{a^2}{2}\mathbb{V}_{i,t}(W_{i,t}) \right\} \right). \tag{7}$$

Using equation (6) and solving for the optimal choice of risky asset yields

$$X_{i,t} = \frac{\mathbb{E}_{i,t}(y_{t+1}) - Rp_t}{a\mathbb{V}_{i,t}(y_{t+1})}. \tag{8}$$

The subscript i in the expectation and the variance operator represents subjective expected value and subjective variance for agent i. The t subscript in the operators indicates that expectation and variance are conditioned at the beginning of time t, that is before the realization of θ_{t+1}. We set net supply of outside share of the risky asset equal to 0 and using the market clearing condition by equating supply to aggregate demand to obtain an implicit pricing equation

$$\sum_{i=1}^{I} X_{i,t} = \sum_{i=1}^{I} \left(\frac{\mathbb{E}_{i,t}(y_{t+1}) - Rp_t}{\mathbb{V}_{i,t}(y_{t+1})} \right) = 0. \tag{9}$$

All agents in the model are assumed to be forward-looking in evaluating the asset price. They expect the asset price to be its fundamental value, which is determined by the present discounted value of the stream of future dividends. This implies that in the model there is only a minimal deviation from rationality, given by the asymmetry in information. This is a crucial aspect and one of the main features that distinguish our paper from other prominent works in the literature. Chiarella (1992), Brock and Hommes (1998) and Lux (1998) among the others, focus on the coexistence of fundamentalists and some type of boundedly rational backward looking agent. We argue that the agents in our model are acting in the optimal way given the information available and that if frictions were removed from the information diffusion process, we would end back in the rational expectations setting. When solving the forward-looking problem, agents in the model assume that other agents behave identically. This assumption leads them to solve the problem as if they were the representative agent. In other words, they consider the aggregate behavior of all agents to be equivalent to their own individual behavior. With this assumption the pricing equation is given by

$$p_t = R^{-1}\mathbb{E}_t\left(y_{t+1}\right), \tag{10}$$

and solving by iterating forward, which is done in section (A) of the Appendix gives

$$p_t = \frac{d}{r} + \frac{\mathbb{E}_t(\theta_{t+1})}{R - \beta}. \tag{11}$$

The first component of equation (11) is the usual discounted value of future expected dividends. The second component is specific of our model and imposed by the persistence of the observable component of dividend θ. Intuitively shocks to this component have an impact on the fundamentals that exponentially decays over time at rate β and need therefore to be discounted in today's price. Having an expression for p_t allow us to compute (see appendix (C) the conditional expectation

$$\mathbb{E}_t(p_{t+1} + d_{t+1}) = \frac{dR}{r} + \frac{R\mathbb{E}_t(\theta_{t+1})}{R-\beta}, \tag{12}$$

and variance

$$\mathbb{V}_t(p_{t+1} + d_{t+1}) = \sigma_\varepsilon^2 + \mathbb{V}_t(\theta_{t+1}) \left(\frac{R}{R-\beta}\right)^2. \tag{13}$$

Hence we can rewrite (9) as

$$\sum_{i=1}^{I} X_{i,t} = \sum_{i=1}^{I} \left(\frac{\frac{dR}{r} + \frac{R\mathbb{E}_{i,t}(\theta_{t+1})}{R-\beta} - Rp_t}{a\left(\sigma_\varepsilon^2 + \mathbb{V}_{i,t}(\theta_{t+1})\left(\frac{R}{R-\beta}\right)^2\right)} \right) = 0, \tag{14}$$

which clarifies that heterogeneity in beliefs is completely related to the component θ_{t+1}. The next section is devoted to describe the mechanism for which agents receive information about this component.

2.2 Information diffusion

Agents are socially connected and are organized in a network. Each agent represents a node, and nodes are entirely characterized by beliefs regarding the observable component of dividends θ_{t+1}. Given the structure of the process, conditional on information available at time t

$$\theta_{t+1} \sim \mathcal{N}\left(\beta\theta_t, \sigma_\eta^2\right).$$

Therefore we model consumers with normally distributed beliefs. Beliefs are heterogeneous since the information set on which agents are conditioning, is agent specific and determined by an endogenous process of information flowing in the network. In each time step, agents have a prior normal distribution regarding this component[1] which depends on the agent's type. There are three possible categories:

- Informed agents. These agents are able to perfectly observe the component before its realization. One can think that this is due to agents having access to privileged or inside information. We prefer to associate this choice with empirical evidence provided by Huberman and Regev (2001) and Peng and Xiong (2006) supporting the idea of different classes of investors. Some agents may possess knowledge to process

[1]Temporarily, we drop the i subscript for convenience, but the process described below is dependent on each agents' position in the network.

domain specific information that other, generalists agent, may lack. For them we have[2]

$$\theta_{t+1} \sim \mathcal{N}\left(\theta_{t+1}, 0\right).$$

- Misinformed agents. These agents imperfectly observe the component before its realization, albeit being not aware of it. For them we have

$$\theta_{t+1} \sim \mathcal{N}\left(\theta_{t+1} + \gamma_{t+1}, 0\right),$$

with $\gamma_{t+1} \sim \mathcal{N}(\mu_\gamma, \sigma_\gamma^2)$, and γ_{t+1} being independent of both η_{t+1} and ε_{t+1}. In the literature there are two main interpretations of misinformation. Theoretically, the focus has been on noise traders, with De Long et al. (1990) being one of the seminal contributions in such framework. Why do individuals trade on noise? As Black (1986) puts it, *"One reason is that they like to do it. Another is that there is so much noise around that they don't know they are trading on noise. They think they are trading on information."* In our case when $\mu_\gamma = 0$, there is no systematic bias in the expectations of these agents. On average they are correct, but they face some noise. The second interpretation which in our model corresponds to $\mu_\gamma \neq 0$ is that these agents trade on and diffuse fake news. In this context Clarke et al. (2021) show that investors are not able to systematically detect fake news even though the reaction of the market is discounted when compered to actual news.

- Uninformed agents. These agents do not observe the component at all, but are aware of the auto-regressive structure and use the last information they have available in taking conditional expectations for their prior

$$\theta_{t+1} \sim \mathcal{N}\left(\beta \mu_{P,t-1}, \sigma_\eta^2\right).$$

with $\mu_{P,t-1}$ being their posterior mean regarding θ_t. The reason for the presence of the posterior in place of the actual realization of θ_t is that uninformed agents are never able to observe individually this components, but will only observe the full dividend realization. The literature provides ample evidence that not all information is immediately processed by investors upon release. The simplest explanation is that of limited attention. Hirshleifer, Lim and Teoh (2009) finds that investor reaction to earnings announcement is weaker in days in which there are multiple simultaneous news. On a similar note Dellavigna and Pollet (2009) show that investors take more time to process news on Friday. Tetlock (2011) shows that investors overreact to stale information, that is information that is similar to previous stories about the same firm and Gilbert et al. (2012) demonstrate that this causes mis-pracing in the

[2]This is in a sense an abuse of notation as a Normal distribution with 0 variance is a degenerate distribution with support at the single point θ_{t+1}, known Dirac's delta function. However this notation is convenient since it will allow us to model the evolution of beliefs of this category of agents in the general framework.

market and are able to construct a profitable strategy exploiting this finding. More recently Blankespoor, deHaan and Marinovic (2020) show that there seems to exist "disclosure processing costs" for which disclosures are not public information as usually defined, but can instead be a form of private information.

The network is static. All edges are exogenously determined and time-invariant. Edges represent information flow between nodes. At the beginning of time t agents have normal prior distribution according to their category. They then receive data in the form of observing node j prior mean if an edge exists between node i and node j. Based on this information they update their beliefs in a Bayesian way. The usual assumption in normal conjugate Bayesian updating is that of known true variance of the likelihood function associated with sampled data. In our case we incorporate a behavioral component in this process, which refers to the evaluation of the precision or accuracy of the source of information. When an agent is connected to another node in the network, we assume that they construct an implicit variance evaluating the forecasting error of the node. Forecasting error is given as the squared difference between the last observable payoff and the payoff prediction implied by source j, that is

$$FE_{j,t} = \left(y_{t-1} - \frac{dR}{r} + \frac{R\mu_{j,t-1}}{R-\beta} \right)^2 \tag{15}$$

To map forecasting error to a comparable variance of the given source, agents multiply their prior variance with the ratio between source j forecasting error and their own

$$\sigma^2_{j,t} = \sigma^2_{0,t} \frac{FE_{j,t}}{FE_{i,t}}$$

that is, if they observe that node j has been more accurate then themselves they will attach higher confidence in their beliefs. They then update their beliefs by a novel mechanism of, which is an extension of Bayes rule to the case of receiving information from K different sources and given in the following proposition.

Proposition 1 (Bayesian Updating of Beliefs) *Assume agents have a normal prior distribution with parameters (μ_0, σ_0^2) and receive K new information $\mu_k, \quad k = 1, 2 \ldots K$. Then agents posterior distribution is normal, with posterior mean given by:*

$$\mu_P = \frac{\sum_{k=0}^{K} \left(\mu_k \cdot [A]_k^{\bar{A}-1} \right)}{\sum [A]^{\bar{A}-1}} \tag{16}$$

and posterior variance:

$$\sigma_P^2 = \frac{\prod_{j=0}^{K} \sigma_j^2}{\sum [A]^{\bar{A}-1}} \tag{17}$$

where, $A = \{\sigma_0^2, \sigma_1^2, \sigma_2^2, \ldots \sigma_K^2\}$, $\bar{A}$ is the cardinality of set A. $[A]^J$ is the set of all distinct combinations of products of size J from set A. $[A]_k^J$ indicates the combination that does not include σ_k^2.

If one takes for example $K = 2$ posterior parameters are given by

$$\mu_P = \frac{\mu_0\sigma_1^2\sigma_2^2 + \mu_1\sigma_0^2\sigma_2^2 + \mu_2\sigma_0^2\sigma_1^2}{\sigma_1^2\sigma_2^2 + \sigma_0^2\sigma_2^2 + \sigma_0^2\sigma_1^2}, \quad \sigma_P^2 = \frac{\sigma_0^2\sigma_1^2\sigma_2^2}{\sigma_1^2\sigma_2^2 + \sigma_0^2\sigma_2^2 + \sigma_0^2\sigma_1^2}.$$

One can see then that the posterior mean is given to a weighted average of the prior means the agent has access to. Moreover each weight given to the signals is equivalent to the Kalman Gain (see Appendix (B) for a discussion. A similar mechanism can be seen as a particular case of the naive updating proposed in Golub and Jackson (2010) where in our case the weights associated to each source j are given by $[A]_j^{\bar{A}-1} / \sum [A]^{\bar{A}-1}$. Moreover the novelty of our approach is in that we are able to simultaneously derive the posterior variance, that while of no importance in their paper, has a fundamental role in the current work, given the risk averse behavior of our agents. Also different in our case is that the information exchange happens only one time per time step, therefore avoiding any potential bias given by repeated information as is the case in DeMarzo, Vayanos and Zwiebel (2003). Of particular interest are then the following situations.

1. An agent considers source k to be absolutely certain. Then in our model we have, $\phi_k = 1$ and

$$\lim_{\sigma_k^2 \to 0} \mu_p = \mu_k \tag{18}$$

$$\lim_{\sigma_k^2 \to 0} \sigma_p^2 = 0 \tag{19}$$

   That is, when an agent uses only one source of information and is totally confident in it, the posterior mean will be equal to the signal, with variance 0.

2. An agent completely disregards source k, corresponding to a situation in which $\phi_k = 0$. Then

$$\lim_{\sigma_k^2 \to +\infty} \mu_p = \frac{\sum_{k=0}^{K} \left(\mu_k \cdot [B]_k^J \right)}{\sum [B]^{\bar{B}-1}} \tag{20}$$

$$\lim_{\sigma_k^2 \to +\infty} \sigma_p^2 = \frac{\prod_{j=0}^{K} \sigma_j^2}{\sum [B]^{\bar{B}-1}} \tag{21}$$

   where $B = A \setminus \sigma_k^2$. When agents believe that a source of information is totally unreliable, their posterior mean and variance will be equal to omitting the source of information.

2.3 Sequence of events

We summarize the model by offering a synthetic visualization of the sequence of events taking place in each time step t in figure (1).

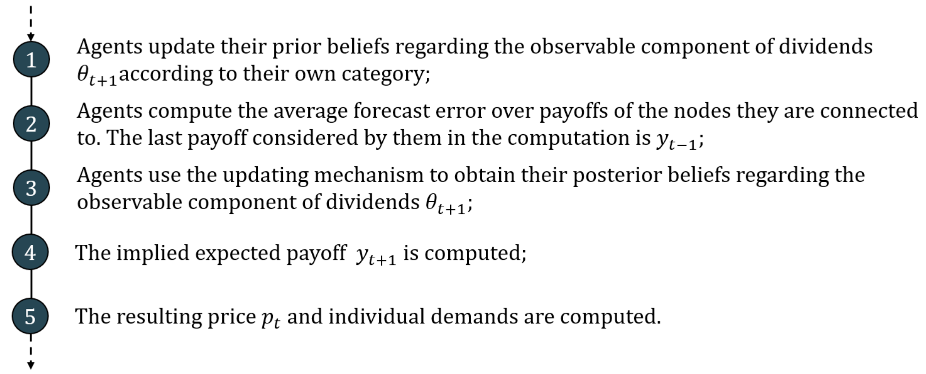

Figure 1: Sequence of events

3 Numerical Simulations

We illustrate the dynamics of the model by numerical simulations. We divide the parameters governing the model behavior into three categories.

Base parameters characterizing the financial market, which are kept constant at the values shown in table (1) in all the scenarios of our analysis.

Table 1: Base Parameters of the Model

Parameter	**Symbol**	**Value**
Total Time Steps	T	500
Number of Agents	I	150
Gross Risk Free Rate	R	1.01
Constant Component of Dividends	d	1.1

Total time steps are sufficient to ensure that behaviors driven by initial conditions are absorbed in the long run. The number of agents in the model is of relative importance only when combined with network specific parameters. Gross risk free rate, and constant component of dividends have an effect only on the level of the resulting price series but not on the quantitative dynamics.

Variable parameters. They are related to the stochastic components of the model and are key factors determining the information diffusion mechanism. Given their relevance we explore their effect on the model for a wide range of values, indicated in table (2).

Table 2: Variable Parameters of the Model

Parameter	Symbol	Range
Volatility of Shocks to the Observable Component of Dividends	σ^2_η	$[0.1, 2.0]$
Mean of the Misinformation Parameter	μ_γ	$[-1.0, 1.0]$
Volatility of the Misinformation Parameter	σ^2_γ	$[0.1, 2.0]$
Volatilty of the Unobservale Component of Dividends	σ^2_ε	$[0.1, 2.0]$
Autocrrelation Coefficient of the Observable Component of Dividends	β	$[0.05, 0.95]$
Proportion of Informed Agents	λ	$[0.01, 0.25]$
Proportion of Misinformed Agents	ξ	$[0.01, 0.25]$

Network parameters. They are specific to the network topology and determines the social structure the agents are embedded in. The analysis is conducted by exploring the effect on the financial market of different structures.

3.1 Small World Network

The first network topology we use is the Watts and Strogatz (1998) Small World Network. We generate the network by starting with a regular lattice of a given degree. We then assign each node to one of the three categories introduced in section (2.2) according to given proportions. Finally we rewire a fraction of edges randomly by a given probability of rewiring, introducing long-distance connections in the network. The main features of such a topology are local clustering, short-average path length and almost homogeneous degree of connection across nodes. The network specific parameters are shown in table (3).

Table 3: Parameters of the Small World Network

Parameter	Range
Network Density	$[0.01, 0.5]$
Probability of Rewiring	$[0.1, 0.5]$

In figure (2) we show key features of the model by using the average over thirty Montecarlo simulations with different stochastic seeds. In panel (a) we plot the network structure used in the simulations. Agents position is randomly determined in the beginning on the experiment and then kept fixed, in order to allow us to track each agent evolution in the different simulations. In panel (b) we report the resulting price of the model and compare it with a benchmark, which we label Representative Agent (RA) Informed price. This is the price implied by having a representative informed agent or a market composed totally by informed agents. Notably, the information diffusion mechanism, guided by the network structure, induces a subtle amplification of volatility. Likewise, it amplifies persistence, as evidenced by an auto-correlation coefficient of 0.83 in contrast to the RA setting's 0.5. This result is driven by the time it takes for shocks to flow into the network. The delay is driven by both an exogenous and an endogenous motif. It is impossible to immediately absorb shocks for nodes without a direct edge to a source of information. These agents are simply precluded real time access to information by the network structure. Agents having this direct connection, should instead endogenously assess the quality and accuracy of it, and only in the limiting case in which they trust it completely, will they immediately incorporate innovations in their beliefs. To study the endogenous flow of information we use panel (c) show a scatter plot of cumulative profits and average forecast error for each agent. Uninformed agents are on average more accurate than the misinformed agents and, maybe surprisingly, also of the informed ones. The reason is the presence of the positive feedback between expectations and future prices we have mentioned. Both informed and misinformed agents act in a dogmatic way not considering how other agents are behaving. This approach leads them to be extremely accurate in forecasting dividends. They however expect prices to always reflect the fundamental value, which is not necessarily the case.In terms of accuracy it pays to be in the majority. Cumulative profits however do not vary proportionally with accuracy. Informed agents are the one profiting the most, followed by misinformed agents. In fact uninformed agents are losing money. The reason can be pinpointed to the risk averse behavior of the investors, which is sparked by the higher uncertainty faced by uninformed investors. Mathematically, the denominator in the individual demand of such kind of investor, which is given by the variance of the future expected payoff is always greater or equal to that of the other categories. This is because the second term in equation (13) is exactly 0 for informed and misinformed agents, and can be 0 for uninformed agents only in the limit when the idiosyncratic component of the dividend process has variance 0. That is, when the only source of variability of dividends is given by the observable component θ, uninformed agents should realize who the informed individuals are, and fully update their beliefs based on their information. Intuitively, uninformed individuals have to be more cautious since the need to take consideration that the source from which they are obtaining information might not be accurate. Lastly we look at the impact on returns. In panel (d) we use a Quantile-quantile (QQ) plot, to identify the presence of fat tails. We compare the simulated quantiles with the theoretical quantiles from a Normal distribution with the same mean and standard deviation of model's returns. There is evidence of lepotkurtosis and we confirm the finding by reporting the kurtosis of the distribution, which is 0.8, in table (6). Panel (e) compares the return distribution with the benchmark case of RA Informed agents. Consistent with our findings, the simulated returns have higher mass around the average and fatter tails.

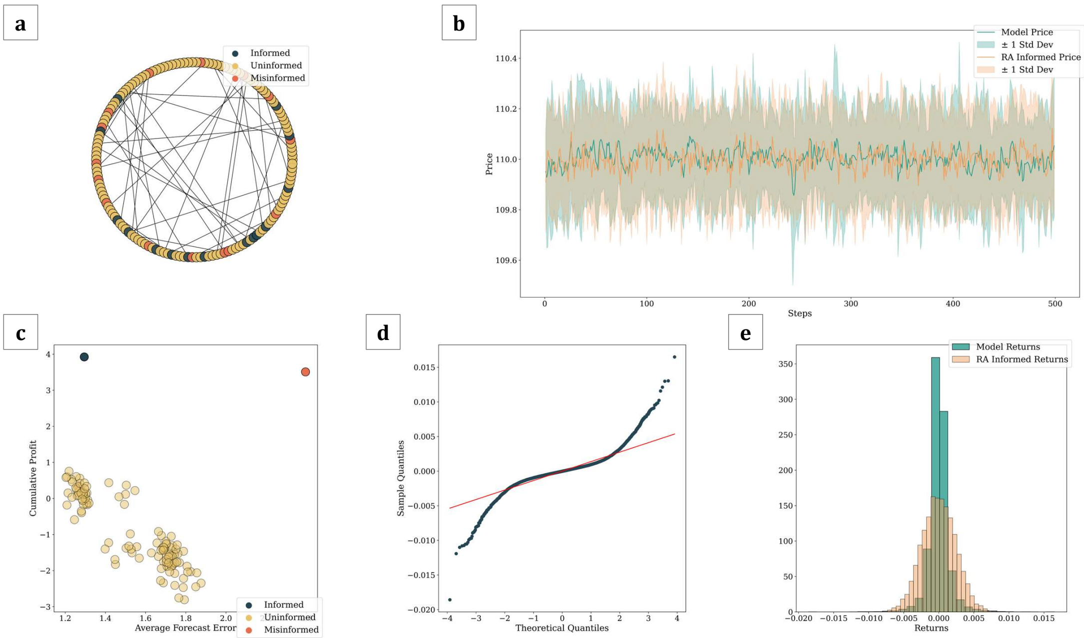

Figure 2: Small World Network

Note: Data are obtained from 30 simulations with different stochastic seeds. **Variable parameters** are: $\sigma_\eta^2 = 0.8$, $\mu_\gamma = 0.0$, $\sigma_\gamma^2 = 1$, $\sigma_\varepsilon^2 = 1$, $\beta = 0.5$, Proportion of Informed Agents = 0.1, Proportion of Misinformed Agents = 0.1. **Network parameters** are: Network Density = 0.05, Probability of Rewiring = 0.1.

3.2 Stochastic Block Network

We now explore the impact of different societies on the financial market. We repeat the analysis by keeping fixed the variable parameters. This allow us to attribute changes in the model dynamics entirely to the network structure. The first scenario we analyze is a completely polarized society. We create it by using a Stochastic Block Model Holland, Laskey and Leinhardt (1983), in which we partition the nodes in order to create two clusters. Informed and misinformed agents are separated and assigned to either the information block or the misinformation one. We denote by density of intra-groups edges, the likelihood of having an edge between agents belonging to the same cluster. This is higher than the density of inter-groups edges, regulating connections between agents belonging to different blocks. Thus not only the network is partitioned, but communication between the two groups is scarce. We report the network specific parameters in table (4).

Table 4: Parameters of the Stochastic Block Network

Parameter	**Range**
Number of Partitions	2
Density of Intra-Groups Edges	[0.1]
Density of Inter-Groups Edges	[0.001]

As before we simulate the model thirty times with different stochastic seeds. Results are displayed in figure (3).

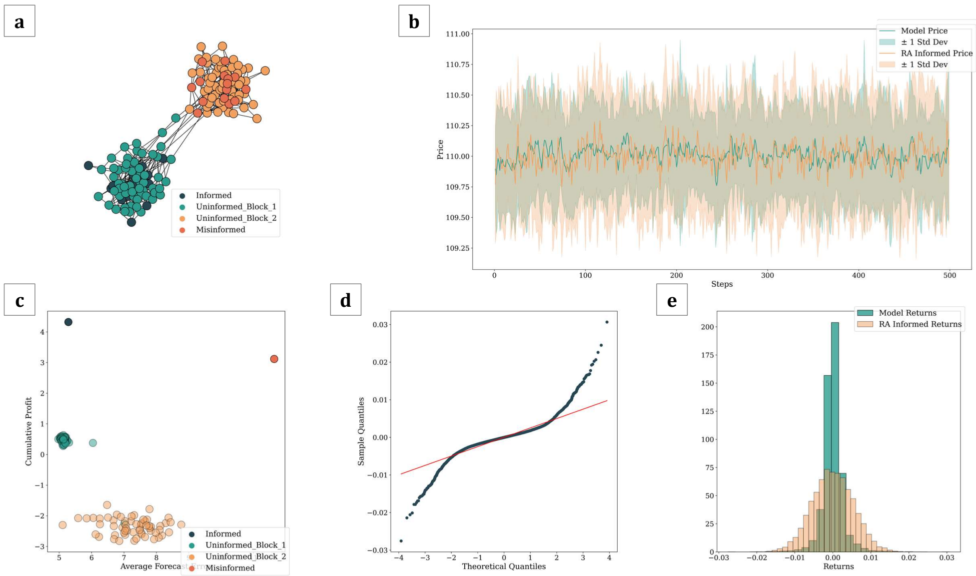

Figure 3: Stochastic Block Network

Note: Data are obtained from 30 simulations with different stochastic seeds. **Variable parameters** are: $\sigma_\eta^2 = 0.8$, $\mu_\gamma = 0.0$, $\sigma_\gamma^2 = 1$, $\sigma_\varepsilon^2 = 1$, $\beta = 0.5$, Proportion of Informed Agents = 0.1, Proportion of Misinformed Agents = 0.1. **Network parameters** are: Number of Partitions = 2, Density of Intra-Groups Edges = 0.1, Density of Inter-Groups Edges = 0.001.

Panel (a) displays the network structure. Each cluster has similar characteristic to the small world structure. Now however uninformed agents have a low chance of being exposed to both information and misinformation. The effect of this segregation is readily apparent in panel (c). Agents belonging to the information block have higher profits than their counterparts in the misinformation block. In panel (b) we see that prices are still exhibiting volatility and persistence amplification, with respect to the RA baseline and also slightly with respect to the small world setting. In table (6) we quantify this effect. Panel (d) and (e) show that Leptokurtosis is now more pronounced although the returns distribution is qualitatively similar.

3.3 Scale Free Network

Until now we have simulated societies in which agents have equal opportunities of sharing their beliefs, given that the average degree was rather homogeneous across the network. For our next scenarios we opt to work with societies in which certain individuals can have a disproportionate impact on society. This concept is similar to the ”guru”, discussed in Tedeschi, Iori and Gallegati (2012). Gurus are are agents that are most imitated by others and can emerge endogenously in the market. The main difference is that in their model, edges are created by a mechanism of preferential attachment based on wealth. Instead we use an exogenous mechanism so by creating a directed[3] Scale-Free Network Bollobás et al. (2003) and forcefully allocating either informed or misinformed agents in the nodes with most outward connections. The network specific parameters are reported in table (5)

Table 5: Parameters of the Scale Free Network

Parameter	**Range**
Network Density	$[0.01, 0.5]$
Probability of Rewiring	$[0.1, 0.5]$

Note: For a detailed explanation of the parameters role we refer to page 2 of Benabou and Laroque (1992).

We begin by analyzing the case of informed agents being the most central. The network topology is displayed in panel (a) of figure (4). Panel (b) shows that volatility amplification is stronger with respect to the the previous undirected cases. Persistence amplification, as reported in table (6) is lower, and the auto-correlation coefficient is 0.68. Panel (c) highlights that informed agents are benefiting from their prominent position in the society, and are earning significantly more then in previous scenarios. Proof of their success in influencing the market is that they are also more accurate in forecasting future payoff. This is because the majority of the network has beliefs, that seems to be distributed around the informed agents one. Errors and beliefs of uninformed agents are however extremely heterogeneous. This is due to many nodes not having a direct connection to the information source, as a result of the scale free property of the network. These individuals are only able to receive news about the observable component of dividends multiple periods after the realization. Even so, it is still better for them to take stale news into consideration, given the persistence of these shocks. This offers a potential explanation to the empirical findings regarding stale information of Gilbert et al. (2012) and Tetlock (2011). The result of this configuration on returns is that of less deviation from the benchmark case and less accentuated leptokurtosis, as we can see in panel (d) and (e).

[3]It must be remarked that although until now we were using undirected networks, informed and misinformed agents were behaving in a dogmatic fashion. This is because a 0 prior variance in their beliefs implies non updating or posterior beliefs exactly equal to the prior.

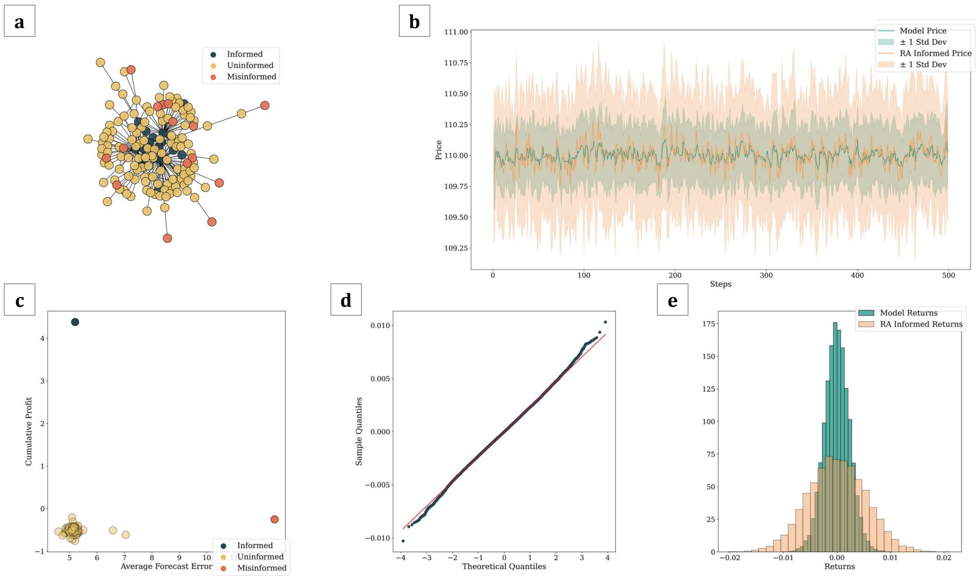

Figure 4: Scale Free Network with Central Informed Agents

We then analyze the second scale free society, in which the most connected nodes are misinformed agents. The network in panel (a) of figure (5) has the same configuration of the previous and the only difference is in the position of agents. The price analysis in panel (b) reveals that this configuration is the one producing the most volatility amplification, with the variance in the simulated model being more than double that of the benchmark case. The series is persistent with an auto-correlation of 0.80, close to the one in the undirected cases. Turning to the profit and accuracy analysis in panel (c) we can see that misinformation has successfully spread into the network. Misinformed agents are the ones with the highest profits and the magnitude of their earnings is more then 4 times higher than in the previous cases. This comes at the expense of uninformed agents who are now on average loosing twice as much as in the previous scenarios. Having in most case connections only to misinformed individuals or other uniformed agents with stale misinformation makes their participation in the market extremely unfruitful. Panel (d) and (e) confirms that returns are again extremely leptokurtic, with kurtosis of 0.79 as we can see in table (6).

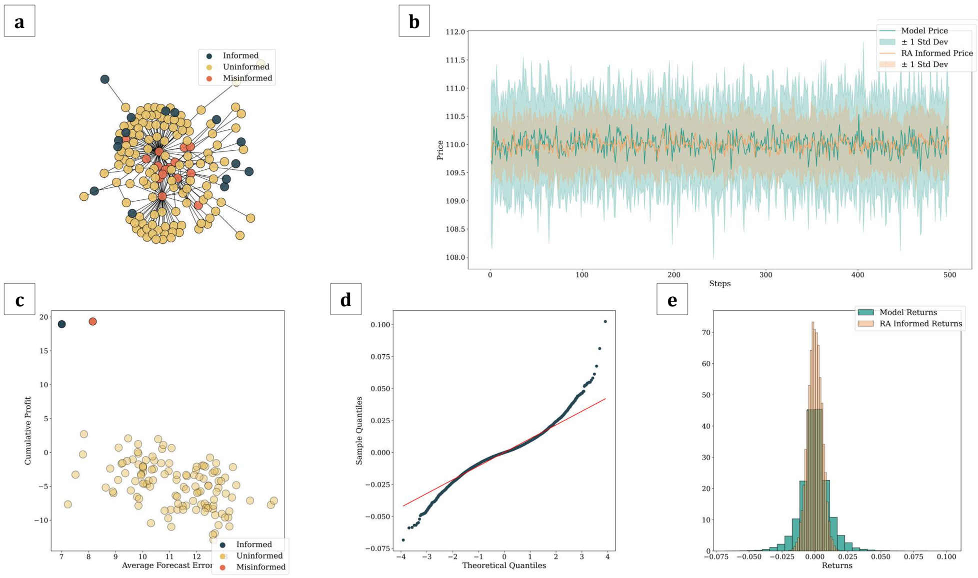

Figure 5: Scale Free Network with Central Misinformed Agents

3.4 Discussion

We now summarize and discuss the results, by collecting important statistics in table (6).

Table 6: Summary of moments for different scenarios

	Variance	**Autocorrelation**	**Kurtosis**
Small World	1.10	0.83	0.81
Stochastic Block Network	1.19	0.84	1.05
Scale Free Informed	1.46	0.68	0.44
Scale Free Misinformed	2.18	0.80	0.79
RA Informed	0.80	0.50	0.00

Contrary to the original Grossman and Stiglitz (1980) model and other works like Benabou and Laroque (1992) in our model communication is individually optimal for two reasons. Given the feedback mechanism, and the presence of misinformed traders, informed individuals are better off revealing their information in the hope of dissuading a larger share of the population to reflect incorrect beliefs into future prices. Moreover as we can see in the case of the scale free networks, the more nodes exists without a direct connection to an information source, the better. This is because when a node incorporate information about the observable component of dividends at a later time, they will push prices towards the direction in which the informed agent has already taken position. To illustrate these point, we provide some analytical results for limiting cases of our model.

We begin the analysis by showing that without misinformed agents the logic of Grossman and Stiglitz (1980) holds.

Proposition 2 (Optimality of no communication) *Assume that the network is fully connected, with no misinformed agents and that uninformed agents prior variance is* ∞[4]*. Then informed agents would earn higher profits by not communicating their beliefs.*

However this argument holds only if every node in the network has a first degree connection to an informed agents. If the network density is not high enough to ensure this property, then it might become more profitable for informed agents to communicate their beliefs. We formalize this in the following proposition.

Proposition 3 (Optimal communication of stale information) *Assume that the network is fully connected, with no misinformed agents and that uninformed agents prior variance is* ∞*. Assume moreover that all but one uninformed agent are two edges away from an informed agent. Then informed agents would earn higher profits by communicating their beliefs.*

We then investigate what parameters are driving the results by Sobol sensitivity analysis Sobol (2001), Saltelli (2002), Saltelli (2010) in section (E). We can see that the two parameters, the auto-correlation coefficient β and the standard deviation of the noise component of the observable component of dividend σ_η are responsible for driving most of the dynamics. To further investigate their role we use contour plot to show the effect of these two parameters on the moments of the resulting price series. For each combination of the parameter in the range $[0.1, 0.8]$ for β and $[0.1, 0.5]$ for σ_η we show the difference between the baseline fully information model and the corresponding simulation in figures (6) to (9).

[4]Indeed this is an extreme and limiting case. It would imply however that the posterior mean of uninformed agents belief would be equal to the one of the informed agents

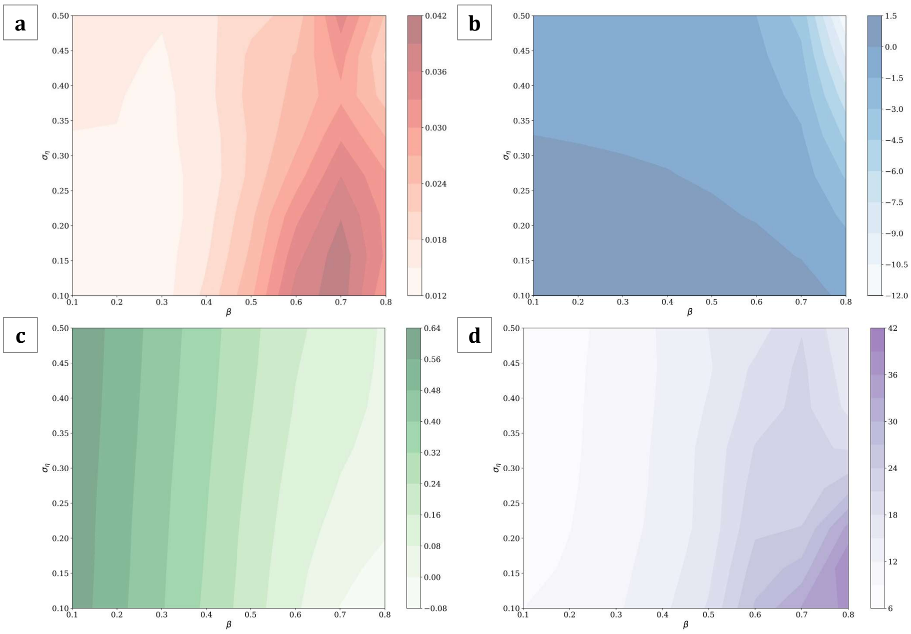

Figure 6: Effect of β and σ_η in the Small World Network

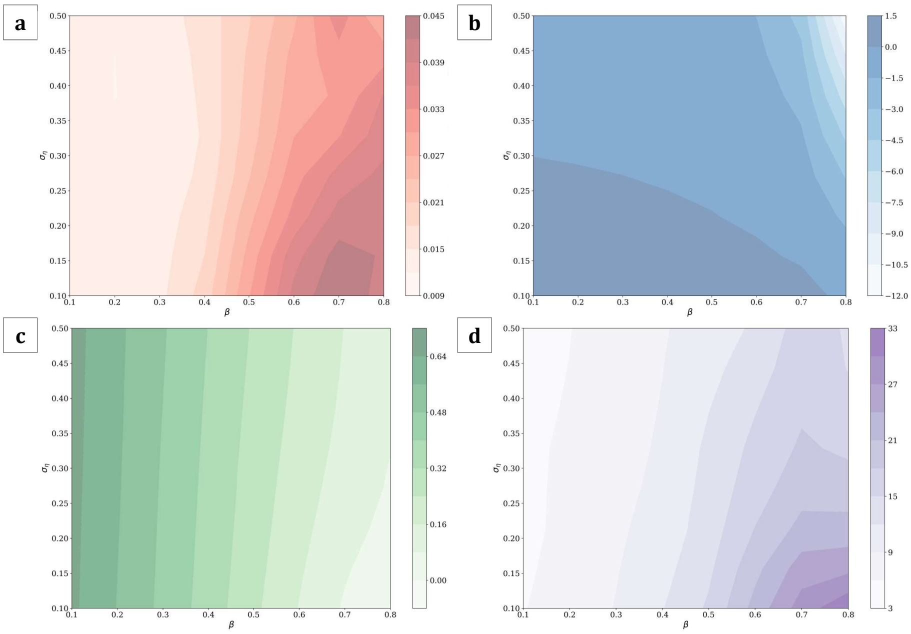

Figure 7: Effect of β and σ_η in the Stochastic Block Model

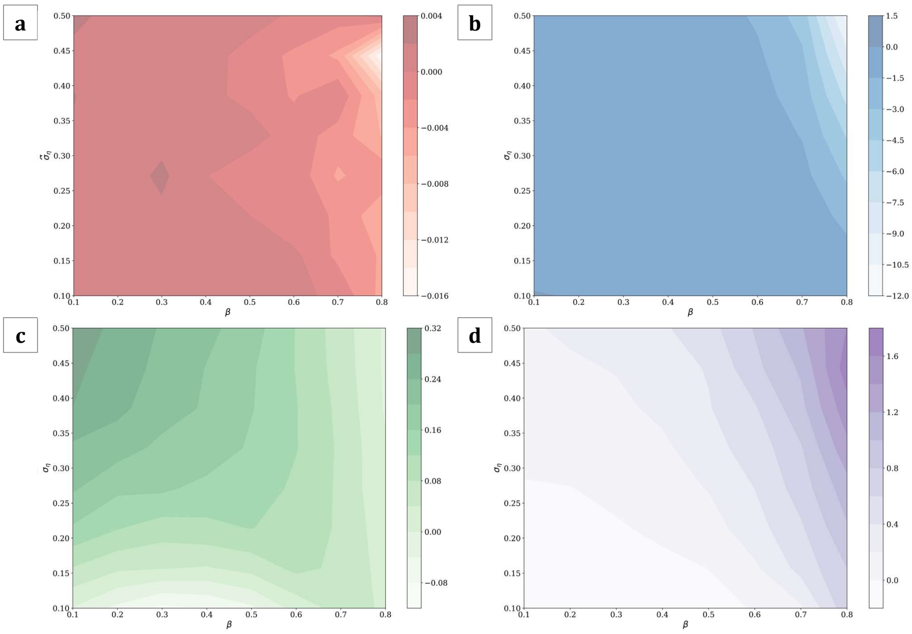

Figure 8: Effect of β and σ_η in the Scale Free Informed Network

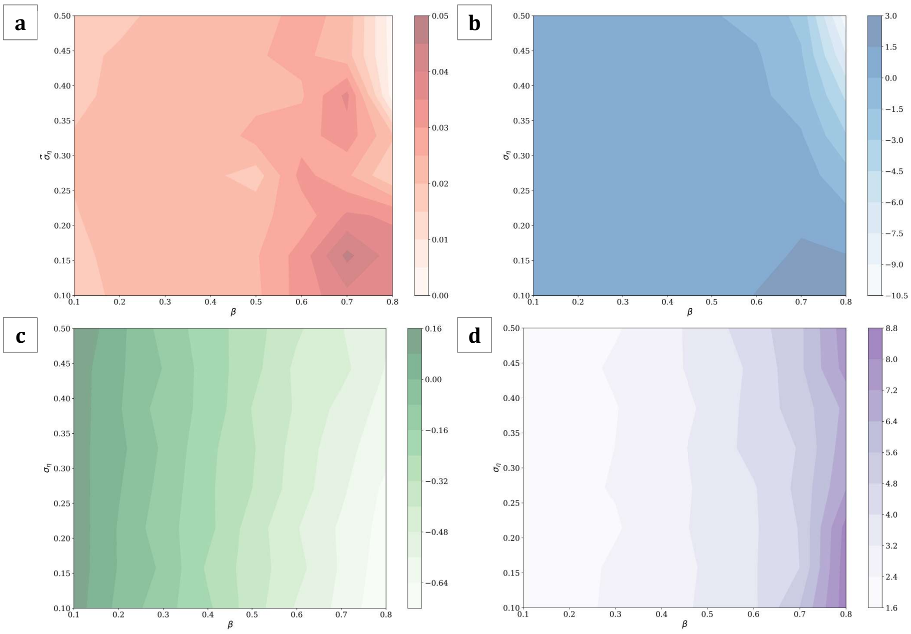

Figure 9: Effect of β and σ_η in the Scale Free Misinformed Network

4 Conclusion

We have presented an agent based model of a financial market to study the interplay between information diffusion and market prices and returns. In this setting agents are connected in a social network and can obtain information from their peers in order to form more accurate forecasts of the underlying dividend process. We proposed a novel mechanism of expectation formation when agents have to evaluate multiple sources of news simultaneously. This is based on Bayesian updating and provides an alternative to perfect rationality, while imposing minimum departures from it. By means of numerical simulations we examined the efficiency implications of multiple social network structures. We demonstrated that empirical features of financial markets, such as excess volatility with respect to fundamentals, persistence amplification and fat tails of returns are emergent behaviors of the system. These features are exacerbated in societies in which misinformed agents occupy prominent positions.

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Appendix

A Forward Looking Price

$$\begin{aligned}
p_t &= R^{-1}\mathbb{E}_t\left(p_{t+1} + d_{t+1}\right) \\
&= R^{-1}\left[\mathbb{E}_t\left(p_{t+1}\right) + \mathbb{E}_t\left(d_{t+1}\right)\right] \\
&= R^{-1}\left[\mathbb{E}_t\left(R^{-1}\mathbb{E}_{t+1}\left(p_{t+2} + d_{t+2}\right)\right) + \mathbb{E}_t\left(d_{t+1}\right)\right] \\
&= R^{-1}\mathbb{E}_t(d_{t+1}) + R^{-2}\mathbb{E}_t(d_{t+2}) + R^{-2}\mathbb{E}_t(p_{t+2}) \\
&= R^{-1}\mathbb{E}_t(d_{t+1}) + R^{-2}\mathbb{E}_t(d_{t+2}) + R^{-2}\mathbb{E}_t(R^{-1}\mathbb{E}_{t+2}\left(p_{t+3} + d_{t+3}\right)) \\
&\vdots \\
&= \sum_{j=1}^{T} R^{-j}\mathbb{E}_t\left(d_{t+j}\right) + R^{-T}\mathbb{E}_t(p_{t+T}),
\end{aligned}$$

which implies, imposing $\lim_{T\to\infty} R^{-T}\mathbb{E}_t(p_{t+T}) = 0$ that in the limit $T \to \infty$

$$p_t = \sum_{j=1}^{\infty} R^{-j}\mathbb{E}_t\left(d_{t+j}\right).$$

Now we focus on the term $\mathbb{E}_t\left(d_{t+j}\right)$. We have that

$$\mathbb{E}_t\left(d_{t+1}\right) = \mathbb{E}_t\left(d + \theta_{t+1} + \varepsilon_{t+1}\right) = d + \mathbb{E}_t\left(\theta_{t+1}\right) + \mathbb{E}_t\left(\varepsilon_{t+1}\right) = d + \mathbb{E}_t\left(\theta_{t+1}\right)$$

[5]

since $\mathbb{E}_t(\varepsilon_{t+1}) = 0$.
Similarly

$$\mathbb{E}_t\left(d_{t+2}\right) = \mathbb{E}_t\left(d + \theta_{t+2} + \varepsilon_{t+2}\right) = d + \mathbb{E}_t\left(\theta_{t+2}\right) + \mathbb{E}_t\left(\varepsilon_{t+2}\right) = d + \beta_t^{\mathbb{E}}\left(\theta_{t+1}\right),$$

since $\mathbb{E}_t(\varepsilon_{t+2}) = 0$ and $\mathbb{E}_t(\theta_{t+2}) = \mathbb{E}_t(\beta\theta_{t+1} + \eta_{t+2})$, and in general

$$\mathbb{E}_t\left(d_{t+j}\right) = d + \beta^{j-1}\mathbb{E}_t\left(\theta_{t+1}\right).$$

Therefore we have

$$p_t = \sum_{j=1}^{\infty} R^{-j}(d + \beta^{j-1}\mathbb{E}_t\left(\theta_{t+1}\right)) = d\sum_{j=1}^{\infty} R^{-j} + \beta^{-1}\mathbb{E}_t\left(\theta_{t+1}\right)\sum_{j=1}^{\infty}\left(\frac{\beta}{R}\right)^j.$$

Now we have

$$d\sum_{j=1}^{\infty} R^{-j} = d\sum_{j=0}^{\infty} R^{-j} - d = d\frac{R}{R-1} - d = \frac{d}{r},$$

[5] Clearly if agents observe the realization of the stochastic component then $\mathbb{E}_t\left(\theta_{t+1}\right) = \theta_{t+1}$. We opt to keep the notation general because this allows us to simultaneously treat also agents that do not observe the realization of this noisy component.

since $|R^{-1}| < 1$. Similarly

$$\beta^{-1}\mathbb{E}_t(\theta_{t+1})\sum_{j=1}^{\infty}\left(\frac{\beta}{R}\right)^j = \beta^{-1}\mathbb{E}_t(\theta_{t+1})\sum_{j=0}^{\infty}\left(\frac{\beta}{R}\right)^j - \beta^{-1}\mathbb{E}_t(\theta_{t+1}) =$$
$$= \beta^{-1}\mathbb{E}_t(\theta_{t+1})\frac{R}{R-\beta} - \beta^{-1}\mathbb{E}_t(\theta_{t+1}) = \beta^{-1}\mathbb{E}_t(\theta_{t+1})\frac{\beta}{R-\beta}.$$

So that finally

$$p_t = \frac{d}{r} + \frac{\mathbb{E}_t(\theta_{t+1})}{R-\beta}.$$

B Relationship of updating to Kalman Filter

Following the notation in chapter 13 of Hamilton (1994) we have the following state space representation of our model

$$\xi_t = F\xi_{t-1} + v_t \quad \text{(state equation)}$$

$$y_t = H\xi_t + w_t \quad \text{(measurement equation)}$$

in which all quantities are scalars and $F \equiv \beta$, $H \equiv 1$. The variance-covariance matrix R associated with w_t is also just the scalar $\sigma^2_{j,t}$. In this contest the a priori variance covariance matrix is just a scalar given by

$$P_{t|t-1} = \mathbb{E}(\xi_t - \hat{\xi}_{t|t-1})^2 = \sigma^2_\eta$$

and the Kalman Gain

$$K_t = P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}$$

collapses to

$$\frac{\sigma^2_\eta}{\sigma^2_\eta + \sigma^2_{j,t}}$$

which is exactly the weight associated to the information received by source j in the case of being connected to source j only.

C Derivation of conditional variance

$$\mathbb{V}_t(p_{t+1} + d_{t+1}) = \mathbb{V}_t(d_{t+1}) + \mathbb{V}_t(p_{t+1}) + 2\mathbb{C}ov_t(d_{t+1}, p_{t+1}).$$

The conditional variance of next period dividends is given by

$$\mathbb{V}_t(d_{t+1}) = \mathbb{V}_t(\theta_{t+1}) + \sigma^2_\varepsilon.$$

The conditional variance of next period price can be derived starting from the expression for p_{t+1} implied by equation (10) and recalling that $\mathbb{E}_{t+1}(\theta_{t+2}) = \beta\theta_{t+1}$. We therefore get

$$\mathbb{V}_t(p_{t+1}) = \mathbb{V}_t\left(\frac{d}{r} + \frac{\beta\theta_{t+1}}{R-\beta}\right) = \mathbb{V}_t(\theta_{t+1})\left(\frac{\beta}{R-\beta}\right)^2.$$

In a similar fashion we can derive the expression for the covariance of future price and dividend as

$$2\mathbb{C}ov_t(d_{t+1}, p_{t+1}) = 2\mathbb{C}ov_t\left(\theta_{t+1}, \frac{\beta\theta_{t+1}}{R-\beta}\right) = 2\mathbb{V}_t(\theta_{t+1})\frac{\beta}{R-\beta}.$$

Rearranging we get equation (13).

D Proofs

D.1 Proof of Proposition(1)

The proof is by induction. First we prove the statement for the base case with only one source, that is $\bar{A} = 2$. This collapses to the normal case of Bayesian updating with conjugate normal prior, therefore we have:

$$\mu_P = \frac{\mu_1\sigma_0^2 + \mu_0\sigma_1^2}{\sigma_0^2 + \sigma_1^2}$$

$$\sigma_P^2 = \frac{\sigma_0^2\sigma_1^2}{\sigma_0^2 + \sigma_1^2}$$

Then we prove that if the statement holds for a generic set A with $\bar{A} = n$, then it holds also for B with $\bar{B} = n+1$. If the statement holds for $\bar{A} = n$, and receive an extra source of information, the new posterior distribution will have parameters:

$$\mu_P = \frac{\mu_{K+1}\frac{\prod_{j=0}^{K}\sigma_j^2}{\sum[A]^{\bar{A}-1}} + \frac{\sum_{k=0}^{K}\left(\mu_k \cdot [A]_k^{\bar{A}-1}\right)}{\sum[A]^{\bar{A}-1}}\sigma_{K+1}^2}{\frac{\prod_{j=0}^{K}\sigma_j^2}{\sum[A]^{\bar{A}-1}} + \sigma_{K+1}^2} = \frac{\frac{\sum_{k=0}^{K+1}\left(\mu_k \cdot \left[A\cup\sigma_{K+1}^2\right]_k^{\bar{A}}\right)}{\sum[A]^{\bar{A}-1}}}{\frac{\prod_{j=0}^{K}\sigma_j^2}{\sum[A]^{\bar{A}-1}} + \frac{\sum\left[\left[A\cup\sigma_{K+1}^2\right]^{\bar{A}}\setminus\prod_{j=0}^{K}\sigma_j^2\right]}{\sum[A]^{\bar{A}-1}}} = \frac{\sum_{k=0}^{K+1}\left(\mu_k \cdot \left[A\cup\sigma_{K+1}^2\right]_k^{\bar{A}}\right)}{\sum\left[A\cup\sigma_{K+1}^2\right]^{\bar{A}}}$$

$$\sigma_P^2 = \frac{\frac{\prod_{j=0}^{K}\sigma_j^2}{\sum[A]^{\bar{A}-1}}\sigma_{K+1}^2}{\frac{\prod_{j=0}^{K}\sigma_j^2}{\sum[A]^{\bar{A}-1}} + \sigma_{K+1}^2} = \frac{\frac{\prod_{j=0}^{K+1}\sigma_j^2}{\sum[A]^{\bar{A}-1}}}{\frac{\prod_{j=0}^{K}\sigma_j^2}{\sum[A]^{\bar{A}-1}} + \frac{\sum\left[\left[A\cup\sigma_{K+1}^2\right]^{\bar{A}}\setminus\prod_{j=0}^{K}\sigma_j^2\right]}{\sum[A]^{\bar{A}-1}}} = \frac{\prod_{j=0}^{K+1}\sigma_j^2}{\sum\left[A\cup\sigma_{K+1}^2\right]^{\bar{A}}}$$

Therefore we have:

$$\mu_P = \frac{\sum_{k=0}^{K+1}\left(\mu_k \cdot [B]_k^{\bar{B}-1}\right)}{\sum[B]^{\bar{B}-1}}$$

$$\sigma_P^2 = \frac{\prod_{j=0}^{K+1} \sigma_j^2}{\sum [B]^{\bar{B}-1}}$$

which are the posterior mean and variance for the set $B = A \cup \sigma_{K+1}^2$ with $\bar{B} = \bar{A} + 1 = n + 1$, hence concluding the proof.

D.2 Proof of Proposition (2)

If there is no communication then all uninformed agents behave in the same way. The resulting price is therefore given by

$$\lambda \frac{\mathbb{E}_{I,t}(y_{t+1}) - Rp_t}{a\mathbb{V}_{I,t}(y_{t+1})} + (1-\lambda) \frac{\mathbb{E}_{U,t}(y_{t+1}) - Rp_t}{a\mathbb{V}_{U,t}(y_{t+1})} = 0,$$

where now the subscripts U and I are used to label the beliefs, homogenous among agents in the same category, of uninformed and informed agents. We then retrieve the beliefs about payoffs for both agents, from equations (12) and (13). For informed agents we have

$$\mathbb{E}_{I,t}(y_{t+1}) = \frac{dR}{r} + \frac{R\theta_{t+1}}{R-\beta}, \quad \mathbb{V}_{U,t}(y_{t+1}) = \sigma_\varepsilon^2,$$

while for uninformed agents we have

$$\mathbb{E}_{U,t}(y_{t+1}) = \frac{dR}{r}, \quad \mathbb{V}_{U,t}(y_{t+1}) = \sigma_\varepsilon^2 (1 + \Phi),$$

where

$$\Phi \equiv \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 (1-\beta^2)} \left(\frac{R}{R-\beta} \right)^2,$$

considering that the unconditional variance of the observable component of dividends is $\mathbb{V}_U(\theta_{t+1}) = \frac{\sigma_\eta^2}{1-\beta^2}$. This implies that we can rewrite the pricing equation as

$$\lambda (1+\Phi) \left(\frac{dR}{r} + \frac{R\theta_{t+1}}{R-\beta} - Rp_t \right) = (\lambda - 1) \left(\frac{dR}{r} - Rp_t \right),$$

and with some manipulations we can get

$$p_t = \frac{d}{r} + \frac{\theta_{t+1}}{R-\beta} \frac{\lambda (1+\Phi)}{1+\lambda\Phi},$$

which by the same reasoning implies

$$p_{t+1} = \frac{d}{r} + \frac{\beta\theta_{t+1} + \eta_{t+2}}{R-\beta} \frac{\lambda (1+\Phi)}{1+\lambda\Phi}.$$

Given the market price we can derive the quantity demanded by informed agents as

$$X_{I,t} = \Lambda \theta_{t+1},$$

with

$$\Lambda \equiv \frac{R(1-\lambda)}{a\sigma_\varepsilon^2(R-\beta)(1+\lambda\Phi)} > 0,$$

since $R > \beta$, $R, a, \sigma_\varepsilon^2$ are all positive and $\left(\frac{1-\lambda}{1+\lambda\Phi}\right) > 0$. This follows from the positivity of Φ and since for $\lambda \in (0,1)$ the function $f(\lambda) = \frac{\lambda(1+\Phi)}{1+\lambda\Phi}$ is a continuous and monotonically decreasing function over the interval $[0,1]$ with $f(0) = 1$ and $f(1) = 0$. We can then compute the excessive profit for the informed agents as

$$\Pi_{I,t} = (y_{t+1} - Rp_t)(\Lambda\theta_{t+1}) = \left(\theta_{t+1}\left(1 - \frac{\lambda(1+\Phi)}{1+\lambda\Phi}\right) + \varepsilon_{t+1} + \eta_{t+2}\frac{\lambda(1+\Phi)}{(R-\beta)(1+\lambda\Phi)}\right)(\Lambda\theta_{t+1}).$$

Finally we take expectations, and recalling that the stochastic components ε_{t+1} and η_{t+2} are Gaussian white noise we get

$$\mathbb{E}(\Pi_{I,t}) = \Lambda\left(\frac{1-\lambda}{1+\lambda\Phi}\right)\mathbb{E}(\theta_{t+1})^2 = \frac{\sigma_\eta^2 R(1-\lambda)^2}{a\sigma_\varepsilon^2(R-\beta)(1+\lambda\Phi)^2(1-\beta^2)} > 0.$$

D.3 Proof of Proposition (3)

Uninformed agents immediately switch to the beliefs of the informed agents, but using θ_t. Then for them we have

$$\mathbb{E}_{U,t}(y_{t+1}) = \frac{dR}{r} + \frac{R\theta_t}{R-\beta}, \quad \mathbb{V}_{U,t}(y_{t+1}) = \sigma_\varepsilon^2.$$

Then the pricing equation is given by

$$p_t = \frac{d}{r} + \lambda\frac{\theta_{t+1}}{R-\beta} + (1-\lambda)\frac{\theta_t}{R-\beta}.$$

Given the market price the quantity demand by informed agents is

$$X_{I,t} = \Gamma(\theta_{t+1} - \theta_t),$$

with

$$\Gamma \equiv \frac{R(1-\lambda)}{a\sigma_\varepsilon^2(R-\beta)}.$$

We compute the excessive profit for the informed agents as

$$\Pi_{I,t} = (y_{t+1} - Rp_t)(\Gamma(\theta_{t+1} - \theta_{t+1})).$$

Focusing on the first term we get

$$(y_{t+1} - Rp_t) = \theta_{t+1} + \varepsilon_{t+1} + \lambda\frac{\beta\theta_{t+1} + \eta_{t+2}}{R-\beta} + (1-\lambda)\frac{\beta\theta_t + \eta_{t+1}}{R-\beta} - \lambda\frac{R\theta_{t+1}}{R-\beta} - (1-\lambda)\frac{R\theta_t}{R-\beta},$$

and rearranging

$$(y_{t+1} - Rp_t) = (1-\lambda)(\theta_{t+1} - \theta_t) + \varepsilon_{t+1} + \lambda \frac{\eta_{t+2}}{R-\beta} + (1-\lambda)\frac{\eta_{t+1}}{R-\beta},$$

$$(y_{t+1} - Rp_t) = (1-\lambda)(\beta-1)\theta_t + \varepsilon_{t+1} + \lambda \frac{\eta_{t+2}}{R-\beta} + (1-\lambda)\left(1 + \frac{1}{R-\beta}\right)\eta_{t+1}.$$

We can also rewrite the demand as

$$X_{I,t} = \Gamma((\beta-1)\theta_t + \eta_{t+1}),$$

so that when taking the expected value we get

$$\mathbb{E}(\Pi_{I,t}) = \Gamma(1-\lambda)(\beta-1)^2 \mathbb{E}(\theta_t)^2 + \Gamma(1-\lambda)\left(1 + \frac{1}{R-\beta}\right)\mathbb{E}(\eta_{t+1})^2,$$

and given that both stochastic components have 0 unconditional mean we have

$$\mathbb{E}(\Pi_{I,t}) = \Gamma(1-\lambda)(\beta-1)^2 \frac{\sigma_\eta^2}{1-\beta^2} + \Gamma(1-\lambda)\left(1 + \frac{1}{R-\beta}\right)\sigma_\eta^2,$$

$$\mathbb{E}(\Pi_{I,t}) = \frac{\sigma_\eta^2 R(1-\lambda)^2}{a\sigma_\varepsilon^2 (R-\beta)}\left(\frac{(\beta-1)^2}{1-\beta^2} + \frac{R-\beta+1}{R-\beta}\right).$$

The final step is that of confronting the expected profits earned by the informed agents in this configuration with the expected profits in the no communication case. We can show that the formers are always greater since

$$\left(\frac{(\beta-1)^2}{1-\beta^2} + \frac{R-\beta+1}{R-\beta}\right) > \frac{1}{(1+\lambda\Phi)^2(1-\beta^2)},$$

since we can rewrite the inequality as

$$(\beta-1)^2 + \frac{(R-\beta+1)(1-\beta^2)}{R-\beta} > \frac{1}{(1+\lambda\Phi)^2}.$$

Now the left-hand side is a monotonically and continuously decreasing function $f(\beta)$ over the interval $(0,1)$ with

$$\lim_{\beta\to 0^+} f(\beta) = \frac{2R+1}{R}, \quad \text{and} \quad \lim_{\beta \to 1^-} f(\beta) = 0,$$

since

$$f'(\beta) = -\frac{\beta^2 - 2R\beta + 2R^2 - 1}{(\beta - R)^2}.$$

The right-hand side is also a monotonically and continuously decreasing function $g(\beta)$ over the interval $(0,1)$ with

$$\lim_{\beta\to 0^+} g(\beta) = \left(1+\lambda\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\right)^{-2}, \quad \text{and} \quad \lim_{\beta\to 1^-} f(\beta) = 0,$$

since

$$g'(\beta) = -\frac{2\left(\frac{2R^2\lambda\sigma_\eta^2}{\sigma_\varepsilon^2(R-\beta)^3(1-\beta^2)} + \frac{2R^2\lambda\sigma_\eta^2\beta}{\sigma_\varepsilon^2(R-\beta)^2(1-\beta^2)^2}\right)}{\left(\frac{R^2\lambda\sigma_\eta^2}{\sigma_\varepsilon^2(R-\beta)^2(1-\beta^2)}+1\right)^3}.$$

Finally since

$$\lim_{\beta\to 0^+} f(\beta) > 1 > \lim_{\beta\to 0^+} g(\beta),$$

we conclude that the inequality holds for every $\beta \in (0,1)$.

E Sensitivity Analysis

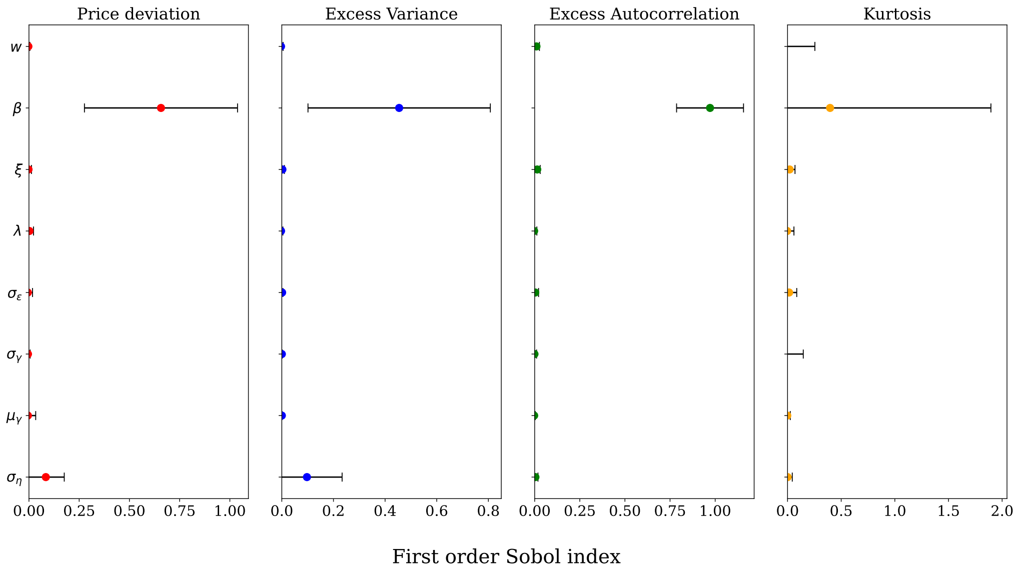

Figure 10: Sensitivity Analysis for the Small World Network

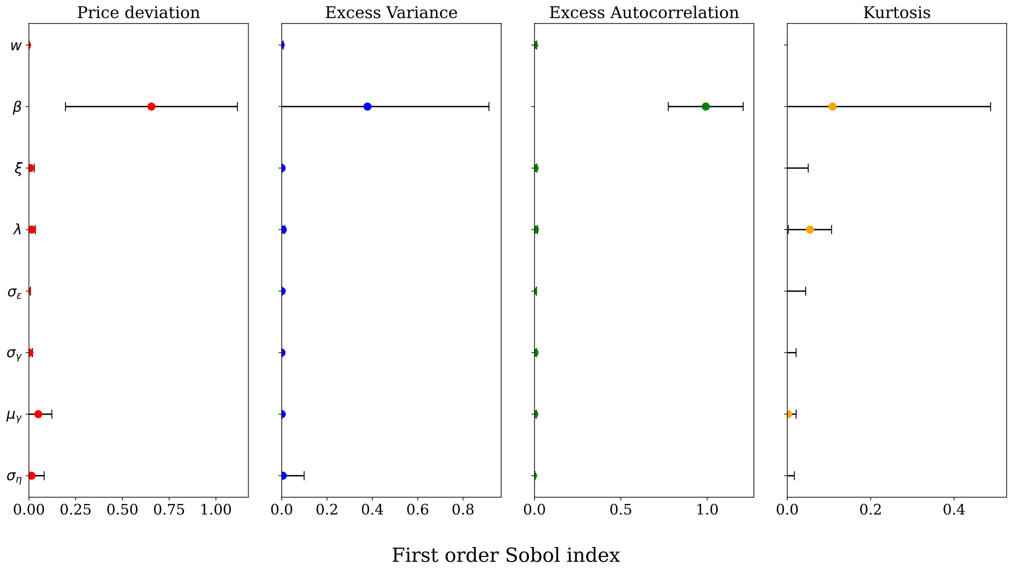

Figure 11: Sensitivity Analysis for the Stochastic Block Model

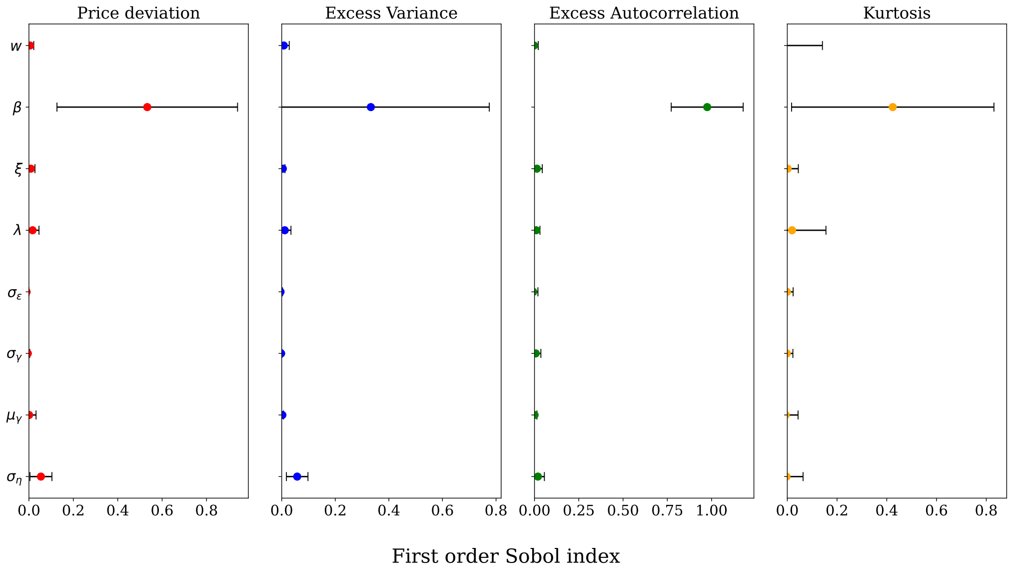

Figure 12: Sensitivity Analysis for the Scale Free Informed Network

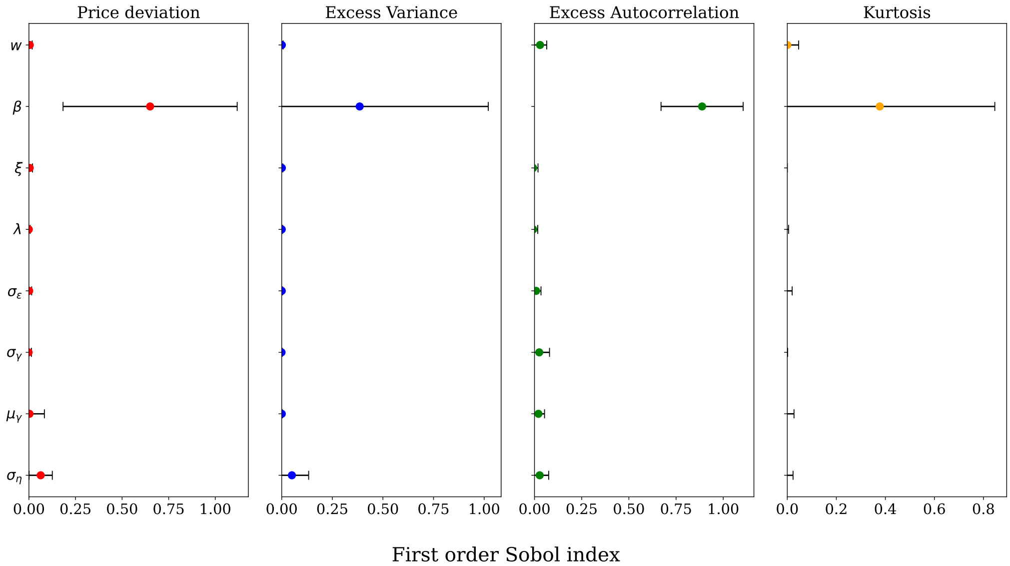

Figure 13: Sensitivity Analysis for the Scale Free Misinformed Network